CAPITAL UNIVERSITY OF SCIENCE AND TECHNOLOGY, ISLAMABAD

Thermal Radiation Effect on an MHD Eyring-Powell Fluid Flow

by

Nabila Riaz

A thesis submitted in partial fulfillment for the degree of Master of Philosophy

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I dedicate my work to my father who had been an inspiration for throughout my life.

CERTIFICATE OF APPROVAL

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by

Nabila Riaz

(MMT171024)

THESIS EXAMINING COMMITTEE

S. No.	Examiner	Name	Organization
(a)	External Examiner	Dr. Muhammad Imran	Gordon College, Rawalpindi
(b)	Internal Examiner	Dr. Shafqat Hussain	CUST, Islamabad
(c)	Supervisor	Dr. Muhammad Sagheer	CUST, Islamabad

Dr. Muhammad Sagheer
Thesis Supervisor
October, 2019

Dr. Muhammad Sagheer	Dr. Muhammad Abdul Qadir
Head	Dean
Dept. of Mathematics	Faculty of Computing
October, 2019	October, 2019

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(Nabila Riaz)

Registration No: MMT171024

Abstract

A two-dimensional magnetohydrodynamics boundary layer flow of the Eyring-Powell fluid on a stretching surface in the presence of thermal radiation and the Joule heating is analyzed. The flow model in the form of the partial differential equations is transformed into a system of non-linear and coupled ordinary differential equations by implementing appropriate similarity transformations. The resulting ordinary differential equations are solved numerically by the shooting method using the computational software MATLAB. The numerical solution obtained for the velocity and temperature profiles has been presented through graphs for different choice of the physical parameters. The skin friction coefficient and the local Nusselt numbers are computed and analyzed. The magnetic field is found to have a direct relation with the temperature profile and an inverse with the velocity profile. Increasing the thermal radiation, the temperature tends to rise.

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Abbreviations

BC	Boundary Condition
BVP	Boundary Value Problem
IVP	Initial Value Problem
MHD	Magnetohydrodynamics
ODEs	Ordinary Differential Equations
PDEs	Partial Differential Equations

Symbols

B_0	Uniform Transverse Magnetic Field
R	Thermal Radiation Parameter
b	Stretching Parameter
Re_x	Local Reynolds Number
C	Concentration of the Species
C_f	Skinfriction Coefficient
C_p	Specific at Constant Pressure
q_r	Radiative Heat Flux
q_w	Rate of Heat Transfer
E_c	Eckert Number
K	Material Parameter
β	Coefficient of the Thermal Expansion
θ	Non-dimensional Temperature
k^*	Absorption Coefficient
M	Magnetic Field Parameter
η	Similarity Variable
σ	Electrical Conductivity
ρ	Density of the Fluid
ν	Kinematic Viscosity
Nu_x	Local Nusselt Number
σ^*	Stefan-Boltzmann Constant
T	Temperature of Fluid

Chapter 1

Introduction

An Eyring-Powell fluid is an important subclass of non-Newtonian fluid. This kind of fluids are further classified into differential types, the integral models and shear rate models. Eyring-Powell fluid has distinct preference over the power-law model and depends on the kinetic theory of liquids. Newtonian behaviors f this fluid decrease for the low and high shear rates. These fluids are quite valuable for their application in many engineering, manufacturing and industrial areas such as pulp, plasma and other biological technology. Besides this, these fluids have essential aspects in fermentation, boiling, bubbles, column, plastic foam processing and in many more with substances such as mud, dyes, toothpaste, blood, corn starch, custard and honey being a few trivial examples of the non- Newtonian fluids. Metzner and Otto [1] focused on the shear rate of non-Newtonian fluids. They worked on the relation of fluid's shear-rate and the speed of the fluids at which they are flowing. Whereas, Rajagopal [2] focused upon the unsteady, incompressible and unidirecional flows of non-Newtonian fluids and later in 1982 with the help of Gupta [3], he found the exact solution for the non-Newtonian fluids . Eldabe et al. [4] elaborated the impact of isochoric non-Newtonian fluid flow by using the Eyring-Powell fluid model. Furthermore, many non-Newtonian fluids discussed by many researchers [5–8].

Boundary layer flow has broad applications in the engineering and industrial fields.

For its broader application in the field of Engineering and manufacturing the boundary layer flow is discussed in detail by Crane [9] with particular emphasis on boundary layer thickness of the stretching sheet. As a consequence of this work, he succeeded in presenting the similarity solution of the two-dimensional flow in the closed form. Bhattacharya [10] examined the stagnation point flow with the brunt of chemical reaction towars the elastic sheet. It was another scientist Hayat et al. [11, 12] who focused his studies upon the convective heat transfer flow of liquid over permeable surface with Newton's boundary-conditions while he conferred the impact of radiation on the flow of an Eyring-Powell fluid in magnetohydrodynamics flows. Javed et al. [13] discussed the Eyring Powell liquid within the nearness of thick boundary layer stream over a stretchable plate. Ara et al. [14] presented the heat transfer flow in Eyring-Powell fluid with the effect of thermal radiation. Khan et al. [15] investigated the heat transfer flow of a fluid with the existence of warm radiation towards a circular stretching surface. Rahimi et al. [16] resorted to the method of collocation for the solution of fluid with the boundary layer flow in an unbounded domain. Sharma et al. [17] discussed the impression of thermal radiation on the liquids towards a stretchable sheet.

The study of the properties of the magnetic field and electrically conducting fluid is known as magnetohydrodynamics. The fundamental examples of magnetofluids are salt water, liquid metals, plasmas and electrolytes. Ibrahim et al. [18] deliberated the consequence of the irresistible field on the stagnation point stream and warm exchange on the nanofluid over a stretchable plate. They remarked that both the surface drag coefficient and the rate of heat transfer are the increasing functions of the speed. Akbar et al. [19] focused on the two-dimensional fluid flow with the effect of magnetic field towards a stretching surface. It was another researcher, Hina [20] who discussed the work on magnetohydrodynamics on the peristaltic motion of Eyring-Powell with the consequence of warm and mass exchange and analyzed that by increment in the warm exchange coefficient, the temperature is also increased whereas the heat transfer coefficient is decreased by an increment in the magnetic field strength. The impact of MHD and nonlinear thermal radiation on an Eyring-Powell fluid over a permeable stretching surface was observed by Bhatti et al. [21]. They analyzed that the fluid motion is decreasing by an increment in the magnetic field. Babu et al. [22] analyzed the mass transfer flow for an Eyring-Powell fluid with the variation of magnetic field on a permeable expanding sheet. Narayana et al. [23] focused on the numerical study of two dimensional Eyring-Powell fluid with the impact of magnetohydrodynamics and heat transfer flow through a linear stretchable surface. Hayat et al. [24] worked on the magnetic nanoparticles for a radiative flow of an Eyring-Powell fluid by including thermophoresis and Brownian motion. The flow is assumed to pass through a stretching cylinder with the effect of convection. The inflence of Biot number on the temperature field has been observed in this article. Mahanthesh et al. [25] inspected the time dependent three-dimensional stream towards a stretchable sheet with the effect of Joule heating and thermal radiation. The shooting technique has been used for computing the numerical results.

1.1 Thesis Contributions

The main objective of this thesis/research work is to study the effect of Joule heating and thermal radiation on magnetohydrodynamics boundary layer flow of an Eyring Powell liquid. The problem is related to two coupled nonlinear PDEs which are converted into a system of coupled ODEs . The numerical technique called the shooting method has been employed to solve the system. Numerical results and graphs are presented to clarify the solutions.

1.2 Outline of Thesis

This thesis is further comprises of four chapters as detailed below:

Chapter 2 contains some important definitions, concepts and laws that are helpful in understanding the present work.

Chapter 3 gives a comprehensive review of Hayat et al. [26]. It presents the numerical analysis of a 2D Eyring-Powell fluid by including convective boundary conditions.

Chapter 4 extends the model explained in Chapter 3 by including the effects of Joule heating and thermal radiation.

Chapter 5 summarizes the research work and gives the main conclusion arising from the whole discussion.

Chapter 2

Fundamental Concepts

In current Chapter, some definitions, basic laws, terminologies, basic concepts and some classical methods for solving nonlinear differential equations would be described.

2.1 Basic Definitions Related to Fluid

Definition 2.1.1. (Fluid)[27]

"A fluid is defined as a substance that deforms continuously when acted on by a shearing stress of any magnitude. A shearing stress (force per unit area) is created whenever a tangential force acts on a surface. "

Definition 2.1.2. (Fluid Mechanics) [28]

"Fluid mechanics is defined as the science that deals with the behavior of fluids at rest (fluid statics) or in motion (fluid dynamics), and the interaction of fluids with solid or other fluids at the boundaries."

Definition 2.1.3. (Fluid Statics) [29]

"It is the field of physics that involves the study of fluids at rest. These fluids are not in motion, that means they have achieved a stable equilibrium state, so fluid statics is largely about understanding these fluid equilibrium conditions."

Definition 2.1.4. (Fluid Dynamics) [29]

"The branch of fluid mechanics that covers the properties of the fluid in the state of progression from one place to another is called fluid dynamics."

Definition 2.1.5. (Viscosity) [29]

"This is internal property of the fluid by virtue of which it offers resistance to the flow. Mathematically viscosity is described as the ratio of the shear stress to the rate of shear strain. i.e,

$$Viscosity(\mu) = \frac{Shear\ stress}{Rate\ of\ shear\ strain}.$$

In above expression μ is called the co-efficient of viscosity. This is also known as the absolute viscosity or simply viscosity having dimensions $[\frac{M}{LT}]$. Water is a thin fluid having low viscosity and on other hand honey is thick fluid carrying higher viscosity. Usually liquids have non-zero viscosity."

Definition 2.1.6. (Kinematic Viscosity) [29]

"The ratio of dynamic viscosity to density, a quantity in which no force is involved. Kinematic viscosity can be obtained by dividing the absolute viscosity of a fluid with the fluid mass density as

$$\nu = \frac{\mu}{\rho}, \tag{2.1}$$

where ρ denote density and ν denote dynamic viscosity respectively."

Definition 2.1.7. (Eyring-Powell Fluid)

"It is some kind of a model of a non-Newtonian fluid that includes plasticity in addition to viscosity and it was published in 1944. Eyring and Powell did some fitting of measured data and came up with some kind of mathematical equation to represent the non-Newtonian behavior of some class of materials with time dependent behavior that depends on the rate of change of shear. "

2.2 Types of Flows

Definition 2.2.1. (Flow) [29]

"It is the deformation of the material under the influence of different forces. If the deformation increase is continuous without any limit then the process is known as flow."

Definition 2.2.2. (Uniform Flow) [29]

"When the velocity of flow does not change either in magnitude or in direction at any point in a flowing fluid, for a given time, it is said to be a uniform flow. In other words, it is the flow of a fluid in which each particle moves along its line of flow with constant speed and the cross section of each stream tube remains unchanged."

Definition 2.2.3. (Non-uniform Flow) [29]

"When there is change in velocity of the flow at different points in a flowing fluid, for a given time, it is said to be non-uniform flow. For example, the flow of liquids under pressure through long pipelines of varying diameter is referred to as non-uniform flow."

Definition 2.2.4. (Compressible Flow) [29]

"The flow in which the material density varies during fluid flow is said to be compressible flow. Compressible fluid flow is used in high-speed jet engines, aircraft, rocket motors also in high-speed usage in a planetary atmosphere, gas pipelines and in commercial fields. Mathematically, it is expressed as

$$\rho(x, y, z, t) \neq c, \tag{2.2}$$

where c is constant."

Definition 2.2.5. (Incompressible Flow)

"A flow where the volume and the density of the flowing fluid remains constant is known as incompressible flow. All liquids are normally supposed to have incompressible flow. Mathematically, it is expressed as

$$\rho(x, y, z, t) = c, \tag{2.3}$$

where c is constant."

Definition 2.2.6. (Steady Versus Unsteady Flows) [29]

"A flow is said to be steady flow in which the fluid properties do not change with time at a specific point, i.e.

$$\frac{\partial \lambda}{\partial t} = 0, \tag{2.4}$$

where λ is any fluid property.

A flow is said to be unsteady flow in which fluid properties change with time. i.e.

$$\frac{\partial \lambda}{\partial t} \neq 0."$$

Definition 2.2.7. (Laminar Versus Turbulent Flows) [28]

"The highly ordered fluid motion characterized by smooth layers of fluid is called laminar. The flow of high-viscosity fluids such as oils at low velocities is typically laminar. The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations is called turbulent. The flow of low viscosity fluid such as air at high velocities is typically turbulent."

Definition 2.2.8. (Viscous and Inviscid Flow) [28]

"Flows in which frictional effects are significant are called viscous flows. However, in many flows of practical interest, there are regions (typically regions not closed to solid surfaces) where viscous forces are negligibly small compared to internal pressure forces in these regions fluid flow is said to be inviscid flow."

2.3 Classification of Fluids

Definition 2.3.1. (Newtonian Versus Non-Newtonian Fluids) [29]

"Fluids in which shear stress is directly proportional to rate of deformation are Newtonian fluids. The term non-Newtonian is used to classify all fluids in which shear stress is not directly proportional to shear rate.

Most common fluids such as water, air, and gasoline are Newtonian under normal conditions. Thus in terms of the coordinates, Newtons law of viscosity is given for one-dimensional flow by

$$\tau_{yx} = \mu \frac{du}{dy}. \tag{2.5}$$

Two familiar examples of non-Newtonian fluids are toothpaste and Lucite paint. The latter is very thick when in the can, but becomes thin when sheared by brushing. Toothpaste behaves as a fluid when squeezed from the tube. However, it does not run out by itself when the cap is removed. There is a threshold or yield stress below which toothpaste behaves as a solid."

2.4 Basic Governing Equations

Definition 2.4.1. (Generalized Continuity Equation) [29]
"Continuity equation is obtained from the law of conservation of mass which states that mass can neither be created nor be destroyed inside a control volume. The mass inside the fixed control system will not change if we examine a differential control volume system enclosed by a surface fixed in space, Then the equation of continuity can be written as

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho V) = 0. \tag{2.6}$$

If the density is constant and spatially uniform, in that case Eq. (2.6) become

$$\nabla.V = 0."$$

Definition 2.4.2. (Generalized Momentum Equation) [29]
"The equation of generalized linear momentum for the fluid particle is acquired. It is expressed as: the net force F acting on a fluid particle is equal to the time rate of change of linear momentum. Consider the mass in a system denoted by control surface of infinitesimally small dimensions dx, dy and dz. The mass of the system is steady.

Newtons second law can be composed as

$$m\frac{DV}{Dt} = F.$$
$$\rho\frac{DV}{Dt} = \nabla.\tau + \rho b,$$

where ρb is the net body force, $\nabla.\tau$ is the surface forces and τ is the Cauchy stress tensor."

Definition 2.4.3. (Conservation of Energy) [28]

"Energy can be transferred to or from a closed system by heat or work, and the conservation of energy principle requires that the net energy transfer to or from a system during a process be equal to the change in energy content of the system. Control volume involves energy transfer via mass flow also, and the conservation of energy principle, also called the energy balance, is expressed as.

$$E_{in} - E_{out} = \frac{dE}{dt}."$$

2.5 Heat Transfer and Related Terms

Definition 2.5.1. (Heat Transfer) [29]

"It is the energy transfer due to the temperature difference. At the point when there is a temperature contrast in a medium or between media, heat transfer must take place. Heat transfer occurs when the temperature of objects are not equal to each other and refers to how this difference is changed to an equilibrium state. Thermodynamics then deals with things that are in the equilibrium state."

Definition 2.5.2. (Conduction) [29]

"Conduction is the process in which heat is transferred through the material between the objects that are in physical contact."

Definition 2.5.3. (Convection) [29]

"Convection is a mechanism in which heat is transferred through fluids (gases or liquids) from a hot place to a cool place."

Definition 2.5.4. (Radiation) [29]

"Radiation is the energy transfer due to the release of photons or electromagnetic waves from a surface volume. Radiation doesn't require any medium to transfer heat. The energy produced by radiation is transformed by electromagnetic waves."

Definition 2.5.5. (Joule Heating) [29]

"The heat which is produced due to flow of current through conductor is called Joule heating."

Definition 2.5.6. (Mass Transfer) [30]

"Mass transfer is the flow of molecules from one body to another when these bodies are in contact or within a system consisting of two components when the distribution of materials is not uniform.e-g When copper plate is placed on steel plate, some molecules from either side will diffuse into the other side. When salt is placed in a glass and water poured over it,after sufficient time the salt molecules will diffuse into water body. Usually mass transfer takes place from a location where the particular component is proportionately low."

Definition 2.5.7. (Magnetohydrodyamics) [29]

"It is the physical mathematical framework that concern the dynamics of magnetic fields in electrically conducting fluids, e.g. in plasma and liquid metals."

Definition 2.5.8. (Viscous Dissipation) [29]

"The process in which the work done by fluid is converted into heat is called viscous dissipation."

Definition 2.5.9. (Boundary Layer) [27]

"One of the great advancements in fluid mechanics occurred in 1904 as a result of the insight of Ludwig Prandtl (1875-1953), a German physicist and aerodynamicist. He conceived of the idea of the boundary layer, a thin region on the surface of a body in which viscous effects are very important and outside of which the fluid behaves essentially as if it were inviscid. Clearly the actual fluid viscosity is the same throughout. only the relative importance of the viscous effects (due to the velocity gradients) is different within or outside of the boundary layer. The purpose of the boundary layer is to allow the fluid to change its velocity from the upstream value of U to zero on the surface. For example, boundary layers form on the surfaces of cars, in the water running down the gutter of the street, and in the atmosphere as the wind blows across the surface of the earth (land or water)."

2.6 Dimensionless Numbers

Definition 2.6.1. (Skin Friction Coefficient)[29]

"Skin friction coefficient occurs between the fluid and the solid surface which leads to slow down the motion of the fluid. The skin friction coefficient can be defined as

$$C_f = \frac{2\pi_w}{\rho U^2}$$

where τ_w denotes the wall shear stress, ρ the density and U the free-stream velocity."

Definition 2.6.2. (Reynolds Number) [29]

"It is a dimensionless number which is used to clarify the different flow behaviors like turbulent or laminar flow. It helps to measure the ratio between inertial force and the viscous force. Mathematically,

$$R_e = \frac{\frac{\rho U^2}{L}}{\frac{\mu U}{L^2}} \Rightarrow R_e = \frac{LU}{\nu}$$

where U denotes the velocity of the fluid with respect to object, L the characteristics length. At low Reynolds number, laminar flow arises where the viscous forces are dominant. At high Reynolds number, turbulent flow arises where the inertial forces are dominant."

Definition 2.6.3. (Prandtl Number)[29]

"It is the ratio between the momentum diffusivity (ν) and thermal diffusivity (α). Mathematically, it can be defined as

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/c_p} = \frac{\mu c_p}{k}, \tag{2.7}$$

where μ represents the dynamic viscosity, C_p denotes the specific heat and k stands for thermal conductivity. The relative thickness of thermal and momentum boundary layer is controlled by Prandtl number. For small Pr, heat distributed rapidly corresponds to the momentum."

Definition 2.6.4. (Rate of Heat Transfer)[29]

"It is the ratio of the convective to the conductive heat transfer to the boundary. Mathematically,

$$Nu = \frac{hL}{k},$$

where h stands for convective heat transfer, L for the characteristics length and k stands for the thermal conductivity."

Definition 2.6.5. (Eckert Number)[29]

"It is the dimensionless number used in continuum mechanics. It describes the relation between flows and the boundary layer enthalpy difference and it is used for characterized heat dissipation. Mathematically,

$$Ec = \frac{u^2}{C_p \nabla T}."$$

Chapter 3

Boundary Layer Flow of an Eyring-Powell Fluid

3.1 Introduction

The numerical study of heat exchange of a 2D boundary layer stream of an incompressible Eyring-Powell fluid over a persistently extending surface is investigated. The speed of stretchiness of the surface is considered as a constant. The flow model showed within the frame of PDEs is changed into a framework of non-linear and coupled ODEs by implementing the appropriate similarity transformations. The resulting ODEs are solved numerically by the shooting strategy utilizing the MATLAB. At the end, the numerical outcomes against various pertinent parameter are portrayed in the form of figures and graphs. To check the reliability of our proposed scheme, numerical comparison with already published literature has been made.

3.2 Mathematical Formulation

Let us assume the 2D, laminar and incompressible flow of an Eyring-Powell fluid moving toward a stretchable sheet. Schematic diagram of the system under investigation is shown in Figure 3.1. The sheet is stretching on x-axis with speed $u_w = bx$ whereas b is a stretching constant.

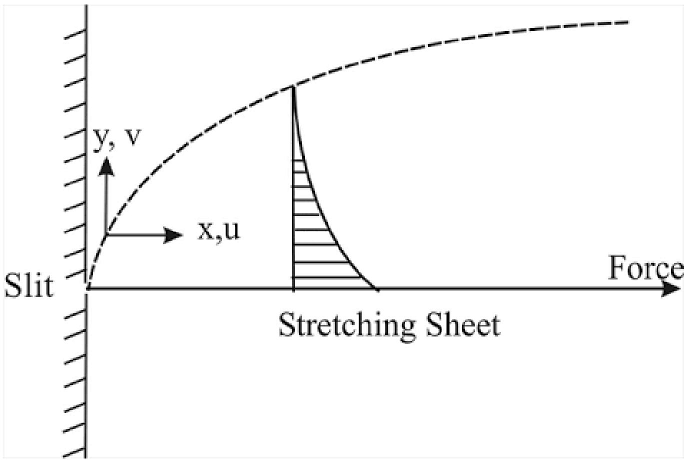

FIGURE 3.1: Geometry for the flow

The governing PDEs are given by.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\nu + \frac{1}{\rho\beta C}\right)\frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho\beta C^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}, \tag{3.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}. \tag{3.3}$$

In equations (3.1)-(3.3), T denotes the temperature, ν the kinematic consistency, ρ the liquid thickness while β and C are the aspects of an Eyring-Powell model. The parameters k represent the warm conductivity and c_p depicts specific heat of liquid, are taken as constants. The associated boundary conditions for the above system have been taken as:

$$\left.\begin{aligned} &u = u_w(x) = bx, v = v_w, -k\frac{\partial T}{\partial y} = h_f(T_f - T) \;\; at \;\; y = 0, \\ &u \to 0, \; T \to T_\infty \; as \; y \to \infty. \end{aligned}\right\} \tag{3.4}$$

The v_w is taken as the mass transfer constant. In case of injection v_w is taken as a positive number while for suction case it is less than zero.

Now the stream function ψ has been introduced which is assumed to satisfy the continuity equation as below. For the conversion of (3.1)-(3.3) into the dimensionless form, the following similarity transformation has been applied:

$$\psi = (a\nu)^{1/2}\, x f(\eta), \tag{3.5}$$

$$\eta = \left(\frac{a}{\nu}\right)^{1/2} y, \tag{3.6}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \tag{3.7}$$

$$T = (T_f - T_\infty)\theta(\eta) + T_\infty. \tag{3.8}$$

For the conversion of (3.1)-(3.3) into a system of ODEs, we use the above transformation. The following steps have been executed for the conversion of the continuity equation.

- $$\begin{aligned} u &= \frac{\partial \psi}{\partial y} \\ &= \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} \\ &= (a\nu)^{1/2}\, x f'(\eta)\left(\frac{a}{\nu}\right)^{1/2} = a x f'(\eta). \end{aligned}$$
- $$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} a x f'(\eta) = a f'(\eta). \tag{3.9}$$
- $$\begin{aligned} v &= -\frac{\partial \psi}{\partial x} \\ &= -\frac{\partial}{\partial x}((a\nu)^{1/2} x f(\eta)) = -(a\nu)^{1/2} f(\eta). \end{aligned}$$
- $$\begin{aligned} \frac{\partial v}{\partial y} &= \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} \\ &= -(a\nu)^{1/2} f'(\eta)\left(\frac{a}{\nu}\right)^{1/2} \\ &= -a f'(\eta). \end{aligned} \tag{3.10}$$

The continuity equation (3.1) is satisfied by using (3.9)-(3.10), as shown below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=af'(\eta)-af'(\eta)=0.$$

Now, we include below the procedure for the conversion of momentum equation (3.2) into the dimensionless form.

- $$u\frac{\partial u}{\partial x}=axf'(\eta)af'(\eta)=a^2xf'^2(\eta). \tag{3.11}$$
- $$\frac{\partial u}{\partial y}=axf''(\eta)\frac{\partial \eta}{\partial y}=axf''(\eta)\left(\frac{a}{\nu}\right)^{1/2}.$$
- $$\begin{aligned} v\frac{\partial u}{\partial y}&=-(a\nu)^{1/2}f(\eta)axf''(\eta)\left(\frac{a}{\nu}\right)^{1/2}.\\ &=-a^2xf(\eta)f''(\eta). \end{aligned}\tag{3.12}$$

To convert the right side of (3.2) into the dimensionless frame, we continue as takes after.

- $$\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(axf''(\eta)\left(\frac{a}{\nu}\right)^{1/2}\right)=x\frac{a^2}{\nu}f'''(\eta).$$
- $$\begin{aligned}\left(\nu+\frac{1}{\rho\beta C}\right)\frac{\partial^2 u}{\partial y^2}&=\left(\nu+\frac{1}{\rho\beta C}\right)\left(\frac{a^2}{\nu}xf'''(\eta)\right)\\ &=\left(1+\frac{1}{\mu\beta C}\right)a^2xf'''(\eta).\end{aligned}\tag{3.13}$$
- $$\begin{aligned}\frac{1}{2\rho\beta C^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}&=\frac{1}{2\rho\beta C^3}\left(axf''(\eta)\left(\frac{a}{\nu}\right)^{1/2}\right)^2\left(\frac{a^2}{\nu}xf'''(\eta)\right)\\ &=\frac{1}{2\beta C^3\mu\nu}\big(a^5x^3(f''(\eta))^2f'''(\eta)\big).\end{aligned}\tag{3.14}$$

Utilizing (3.11)-(3.14), the dimensionless form of (3.2) becomes:

$$\begin{aligned} a^2x((f'(\eta))^2-f(\eta)f''(\eta))&=a^2x\Bigg(\left(1+\frac{1}{\mu\beta C}\right)f'''(\eta)\\ &\qquad-\frac{1}{2\beta C^3\mu\nu}(a^3x^2(f''(\eta))^2f'''(\eta))\Bigg)\\ \Rightarrow (f'(\eta))^2-f(\eta)f''(\eta)&=\left(1+\frac{1}{\mu\beta C}\right)f'''(\eta)-\frac{1}{2\beta C^3\mu\nu}\left(a^3x^2(f''(\eta))^2f'''(\eta)\right).\\ \Rightarrow (f'(\eta))^2-f(\eta)f''(\eta)&=(1+\varepsilon)\,f'''(\eta)-\varepsilon\delta(f''(\eta))^2f'''(\eta). \end{aligned}\tag{3.15}$$

Now, further proceedings for the conversion of (3.3) into the dimensionless form, are:

- $$\frac{\partial T}{\partial x} = \theta'(\eta)(T_f - T_\infty)\frac{\partial \eta}{\partial x} = 0. \tag{3.16}$$
- $$u\frac{\partial T}{\partial x} = 0. \tag{3.17}$$
- $$\begin{aligned} \frac{\partial T}{\partial y} &= \theta'(\eta)(T_f - T_\infty)\frac{\partial \eta}{\partial y} \\ &= (T_f - T_\infty)\sqrt{\frac{a}{\nu}}\ \theta'(\eta). \end{aligned} \tag{3.18}$$
- $$\begin{aligned} v\frac{\partial T}{\partial y} &= -(a\nu)^{1/2}(T_f - T_\infty)\left(\frac{a}{\nu}\right)^{1/2} f(\eta)\theta'(\eta). \\ &= -a(T_f - T_\infty)f(\eta)\theta'(\eta). \end{aligned} \tag{3.19}$$
- $$\begin{aligned} \frac{\partial^2 T}{\partial y^2} &= (T_f - T_\infty)\left(\frac{a}{\nu}\right)^{1/2}\frac{\partial \eta}{\partial y}\theta''(\eta) \\ &= \frac{a}{\nu}(T_f - T_\infty)\theta''(\eta). \end{aligned} \tag{3.20}$$

Using equation (3.17), (3.19) and (3.20) in equation (3.3), we get

$$\begin{aligned} &-a(T_f - T_\infty)f(\eta)\theta'(\eta) = \frac{k}{\rho c_p}\frac{a}{\nu}(T_f - T_\infty)\theta''(\eta). \\ &\Rightarrow \frac{k}{\nu\rho c_p}\theta''(\eta) + f\theta'(\eta) = 0. \\ &\Rightarrow \theta''(\eta) + Prf\theta'(\eta) = 0. \end{aligned} \tag{3.21}$$

The mathematical procedure of the conversion of the dimensional boundary conditions (3.4) into the dimensionless form, is portrayed as.

- $$\begin{aligned} & u(x,y) = bx \quad \text{at} \quad y = 0. \\ \Rightarrow \quad & axf'(\eta) = bx \quad \text{at} \quad \eta = 0. \\ \Rightarrow \quad & f'(0) = \frac{b}{a} = \alpha. \end{aligned}$$
- $$\begin{aligned} & v = v_w \quad \text{at} \quad y = 0. \\ \Rightarrow \quad & -\sqrt{a\nu}f(\eta) = v_w \quad \text{at} \quad \eta = 0. \\ \Rightarrow \quad & f(0) = \frac{v_w}{\sqrt{a\nu}} = S. \end{aligned}$$

- $$-k\frac{\partial T}{\partial y} = h_f(T_f - T) \text{ at } y = 0.$$

$$\Rightarrow \quad -k(T_f - T_\infty)\sqrt{\frac{a}{\nu}}\theta'(\eta) = h_f(T_f - T) \text{ at } \eta = 0.$$

$$\Rightarrow \quad \theta'(\eta) = -\frac{h_f}{k}\sqrt{\frac{\nu}{a}}\left(\frac{T_f - T}{T_f - T_\infty}\right) \text{ at } \eta = 0.$$

$$\Rightarrow \quad \theta'(\eta) = -\frac{h_f}{k}\sqrt{\frac{\nu}{a}}\left(\frac{T_f - T_\infty}{T_f - T_\infty} - \frac{T - T_\infty}{T_f - T_\infty}\right) \text{ at } \eta = 0.$$

$$\Rightarrow \quad \theta'(\eta) = -\gamma(1 - \theta(\eta)) \text{ at } \eta = 0.$$

$$\Rightarrow \quad \theta'(0) = -\gamma(1 - \theta(0)).$$

- $$u(x, y) \to 0 \text{ as } y \to \infty.$$

$$\Rightarrow \quad axf'(\eta) \to 0 \text{ as } \eta \to \infty.$$

$$\Rightarrow \quad f'(\eta) \to 0 \text{ as } \eta \to \infty.$$

- $$T(x, y) \to T_\infty \text{ as } y \to \infty.$$

$$\Rightarrow \quad (T_f - T_\infty)\theta(\eta) + T_\infty \to T_\infty \text{ as } \eta \to \infty.$$

$$\Rightarrow \quad (T_f - T_\infty)\theta(\eta) \to 0 \text{ as } \eta \to \infty.$$

$$\Rightarrow \quad \theta(\eta) \to 0 \text{ as } \eta \to \infty.$$

The final dimensionless form of the governing model is

$$(1 + \varepsilon) f''' - \varepsilon\delta f''^2 f''' - f'^2 + ff'' = 0, \tag{3.22}$$

$$\theta'' + Prf\theta' = 0. \tag{3.23}$$

The associated BCs (3.4) get the form:

$$\left.\begin{aligned} &f(\eta) = S,\ f'(\eta) = \alpha,\ \theta'(\eta) = -\gamma(1 - \theta(\eta)) \text{ at } \eta = 0,\\ &f'(\eta) \to 0,\ \theta(\eta) \to 0 \text{ as } \eta \to \infty. \end{aligned}\right\} \tag{3.24}$$

Different parameters used in the above equations have the following formulation:

$$\left.\begin{aligned} &\varepsilon = \frac{1}{\mu\beta C}, \quad \delta = \frac{a^3x^2}{2C^2\nu}, \quad Pr = \frac{\mu c_p}{k},\\ &S = -\frac{v_w}{\sqrt{a\nu}}, \quad \alpha = \frac{b}{a}, \quad \gamma = \frac{h_f}{k}\sqrt{\frac{\nu}{a}}. \end{aligned}\right\} \tag{3.25}$$

The surface drag coefficient, is defined as:

$$C_{fx} = \frac{\tau_w}{\rho u_w^2}, \tag{3.26}$$

where the surface drag coefficient at the wall is given by

$$\tau_w = \left(\mu + \frac{1}{\beta C}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0} - \frac{1}{6\beta}\left(\frac{1}{C}\left(\frac{\partial u}{\partial y}\right)_{y=0}\right)^3.$$

The surface drag coefficient in the dimensionless form is given by

$$\begin{aligned}
C_{fx} &= \frac{1}{\rho u_w^2}\left(\left(\mu + \frac{1}{\beta C}\right)ax\sqrt{\frac{a}{\nu}}f''(0) - \frac{1}{6\beta}\left(\frac{1}{C}ax\sqrt{\frac{a}{\nu}}f''(0)\right)^3\right) \\
&= \frac{1}{\frac{\mu}{\nu}(ax)^2}\left(\left(\mu + \frac{1}{\beta C}\right)ax\sqrt{\frac{a}{\nu}}f''(0) - \frac{1}{6\beta}\left(\frac{1}{C}ax\sqrt{\frac{a}{\nu}}f''(0)\right)^3\right) \\
&= \frac{\nu ax\sqrt{\frac{a}{\nu}}}{(ax)^2}\left(\left(1 + \frac{1}{\mu\beta C}\right)f''(0) - \frac{a^3x^2}{6\beta\mu\nu C^3}(f''(0))^3\right) \\
&= \sqrt{\frac{\nu}{ax^2}}\left(\left(1 + \frac{1}{\mu\beta C}\right)f''(0) - \frac{a^3x^2}{6\beta\mu\nu C^3}(f''(0))^3\right) \\
&= (Re_x)^{-1/2}\left((1+\varepsilon)f''(0) - \frac{1}{3}\varepsilon\delta(f''(0))^3\right).
\end{aligned}$$

$$\Rightarrow (Re_x)^{1/2}\, C_f x = (1+\varepsilon)\, f''(0) - \frac{1}{3}\varepsilon\delta(f''(0))^3. \tag{3.27}$$

where the Reynolds number is given as

$$Re_x = \frac{ax^2}{\nu}.$$

The rate of heat transfer Nu_x is characterized by:

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \tag{3.28}$$

where

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}.$$

The dimensionless form of the Nu_x, is

$$\begin{aligned} Nu_x &= -\frac{kx(T_f - T_\infty)\sqrt{\frac{a}{\nu}}\theta'(0)}{k(T_w - T_\infty)} \\ &= -x\sqrt{\frac{a}{\nu}}\theta'(0) \\ &= -(Re_x)^{1/2}\theta'(0). \\ \Rightarrow (Re_x)^{-1/2}Nu_x &= -\theta'(0). \end{aligned} \tag{3.29}$$

3.3 Solution Methodology

In our work we used shooting method to solve the transformed ODEs subject to the boundary condition. For the implementation of the shooting method we first convert the system of equations into first order ODEs as follows:

$$f''' = \frac{f'^2 - ff''}{1 + \varepsilon - \varepsilon\delta f''^2}, \tag{3.30}$$

$$\theta'' = -Prf\theta'. \tag{3.31}$$

Let us use the notations:

$$\begin{aligned} &f = y_1,\ f' = y_1' = y_2,\ f'' = y_2' = y_3, \\ &\theta = y_4,\ \theta' = y_4' = y_5. \end{aligned}$$

The resulting initial value problem takes the following shape:

$$\begin{aligned} &y_1' = y_2, && y_1(0) = S, \\ &y_2' = y_3, && y_2(0) = \alpha, \\ &y_3' = \frac{y_2^2 - y_1y_3}{1 + \varepsilon - \varepsilon\delta y_3^2}, && y_3(0) = p, \\ &y_4' = y_5, && y_4(0) = q, \\ &y_5' = -Pry_1y_5, && y_5(0) = -\gamma(1 - q). \end{aligned}$$

Because the numerical computations can not be performed on an unbounded domain, therefore the domain of the above problem has been taken as $[0, \eta_\infty]$ instead of $[0, \infty)$, where η_∞ is a finite positive number for which the variations in the solution are negligible after $\eta = \eta_\infty$. The ideal missing conditions p and q are assumed to satisfy the following relations.

$$y_2(\eta_\infty, p, q) = 0, \tag{3.32}$$

$$y_4(\eta_\infty, p, q) = 0. \tag{3.33}$$

The system of equations (3.32)-(3.33) will be solved by the Newton's method governed by the following formulation.

$$\begin{pmatrix} p^{(k+1)} \\ q^{(k+1)} \end{pmatrix} = \begin{pmatrix} p^k \\ q^k \end{pmatrix} - \begin{pmatrix} \frac{\partial y_2}{\partial p} & \frac{\partial y_2}{\partial q} \\ \frac{\partial y_4}{\partial p} & \frac{\partial y_4}{\partial q} \end{pmatrix}^{-1}_{(p^k,\ q^k,\ \eta_\infty)} \begin{pmatrix} y_2 \\ y_4 \end{pmatrix}_{(p^k,\ q^k,\ \eta_\infty)}.$$

The following new variables have been introduced for the further proceedings.

$$\frac{\partial y_1}{\partial p} = y_6, \quad \frac{\partial y_2}{\partial p} = y_7, \quad \frac{\partial y_3}{\partial p} = y_8, \quad \frac{\partial y_4}{\partial p} = y_9, \quad \frac{\partial y_5}{\partial p} = y_{10},$$
$$\frac{\partial y_1}{\partial q} = y_{11}, \quad \frac{\partial y_2}{\partial q} = y_{12}, \quad \frac{\partial y_3}{\partial q} = y_{13}, \quad \frac{\partial y_4}{\partial q} = y_{14}, \quad \frac{\partial y_5}{\partial q} = y_{15}.$$

By inserting these notations in the above Newton's iterative formulation, we get the form:

$$\begin{pmatrix} p^{(k+1)} \\ q^{(k+1)} \end{pmatrix} = \begin{pmatrix} p^k \\ q^k \end{pmatrix} - \begin{pmatrix} y_7 & y_{12} \\ y_9 & y_{14} \end{pmatrix}^{-1}_{(p^k,\ q^k,\ \eta_\infty)} \begin{pmatrix} y_2 \\ y_4 \end{pmatrix}_{(p^k,\ q^k,\ \eta_\infty)}.$$

Now differentiate the above system of ODEs, first w.r.t p and then w.r.t q. The complete IVP can be written as:

$$
\begin{aligned}
y_6' &= y_7, && y_6(0) = 0,\\
y_7' &= y_8, && y_7(0) = 0,\\
y_8' &= \frac{y_2^2 - y_1 y_3}{(1+\varepsilon-\varepsilon\delta y_3^2)^2}(2\delta\varepsilon y_3 y_8) + \frac{2y_2y_7 - y_1y_8 - y_6y_3}{1+\varepsilon-\varepsilon\delta y_3^2}, && y_8(0) = 1,\\
y_9' &= y_{10}, && y_9(0) = 0,\\
y_{10}' &= -Pr(y_6y_5 + y_1y_{10}), && y_{10}(0) = 0,\\
y_{11}' &= y_{12}, && y_{11}(0) = 0,\\
y_{12}' &= y_{13}, && y_{12}(0) = 0,\\
y_{13}' &= \frac{y_2^2 - y_1 y_3}{(1+\varepsilon-\varepsilon\delta y_3^2)^2}(2\delta\varepsilon y_3 y_{13}) + \frac{2y_2y_{12} - y_1y_{13} - y_{11}y_3}{1+\varepsilon-\varepsilon\delta y_3^2}, && y_{13}(0) = 0,\\
y_{14}' &= y_{15}, && y_{14}(0) = 1,\\
y_{15}' &= -Pr(y_{11}y_5 + y_1y_{15}), && y_{15}(0) = \gamma.
\end{aligned}
$$

The Newton's iterative process is repeated up till the following condition is met.

$$
max\{|y_2(\eta_\infty)|\ ,\ |y_4(\eta_\infty)|\} \leq \xi,
$$

where ξ is a small positive number. For the computational purpose, ξ has been given the value $\xi = 10^{-8}$ whereas η_∞ is set as 8.

3.4 Tabular and Graphical Results

The objective of this area is to analyze the numerical outcomes that are displayed in the forms of tables and graphs. The computations are carried out to analyze the influence of the speed slip parameter δ, proportion parameter α, Prandtl number Pr, Biot number γ, material fluid parameter ε and suction parameter S on the fluid motion and temperature conveyances. Table 3.1 analyzes the numerical values of rate of heat exchange and surface drag coefficient which are computed by the shooting method. For this table, computations are carried out for distinctive values of the criterion ε, Pr, γ, δ and S. The initial guess for the unknown initial condition p, can be chosen from the interval I_f displayed in the table, whereas for the choice of q, very high flexibility is experienced. By increasing the values of ε, Pr, S, γ, both the drag coefficient and heat transfer rate increase while these values decrease with an increment in δ.

ε	δ	Pr	S	γ	$Re_x^{1/2}C_f$	I_f	$Re_x^{-1/2}Nu_x$
0.2	0.2	1	0.5	1	0.5319218	[-0.5, 0.0]	0.4395811
0.3					0.5470097	[-0.4, 0.0]	0.4413199
0.5					0.5757817	[-0.4, 0.0]	0.4442502
1					0.6413143	[-0.3, 0.1]	0.4494621
0.3	0.1				0.5471547	[-0.4, 0.1]	0.4413372
	0.5				0.5465719	[-0.4, 0.0]	0.4412674
	1				0.5458323	[-0.4,-0.1]	0.4411786
	0.2	0.5			0.5470097	[-0.4, 0.0]	0.3081747
		1.2			0.5470097	[-0.4, 0.0]	0.4791642
		1.5			0.5470097	[-0.4, 0.0]	0.5257594
		1	0.2		0.4564414	[-0.3,-0.1]	0.3608930
			1		0.7239605	[-0.6, 0.3]	0.5472029
			1	0.5	0.7239605	[-0.6, 0.3]	0.3536723
				1.5	0.7239605	[-0.6, 0.3]	0.6692802

TABLE 3.1: Numerical outcomes of $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ for distinctive parameters when α =0.5

A comparative review can be seen through different figures. Figures 3.2 and 3.3 highlights the affect of the extending proportion parameter α on the mass fraction field. By these figures, it is obvious that increasing α guides to an increase within the compactness of the speed boundary layer. For $\alpha > 0$, wall-stretchiness and for $\alpha < 0$, wall-shrinking is expected to happen whereas for α=0, there is neither stretching nor shrinking. Figures 3.4 and 3.5 show the influence of S on the fluid motion $f'(\eta)$. The suction parameter S and the fluid motion $f'(\eta)$ are observed to have an inverse relation. An increase in S makes the movement of liquid to diminish.

Figures 3.6 and 3.7 depicts the impact of the material fluid parameter ε and the speed slip parameter δ on the speed field $f'(\eta)$. Figure 3.6 appears that by expanding the values of the material liquid parameter ε, there's an increase within the boundary layer thickness of the speed profile. Figure 3.7 shows an inverse relation between δ and $f'(\eta)$. By expanding the values of δ the boundary layer thickness goes to diminish.

Figures 3.8-3.14 show the effects of different parameters, Pr, α, S, γ, ε and δ on the temperature field. From figures 3.8 and 3.9, it is cleared that as we increment the terms of Prandtl number, the warm diffusivity begins diminishing. So the temperature and the warm boundary layer density are the diminishing capacities of Pr. Figures 3.10 and 3.11 show the influence of S on $\theta(\eta)$. The temperature goes to decrease as we increase the values of S. Figures 3.12 and 3.13 indicate the impact of Biot number γ on temperature. From both of these figures, it is seen that as we increment the values of Biot number, the rate of warm exchange too increments. So for an increase within the heat exchange rate, the temperature moreover increments. Figure 3.14 gives the affect of ε and δ on the temperature distributions. By increasing the values of these material fluid parameters, the temperature goes to decrease. Both of these have an inverse relation with the temperature.

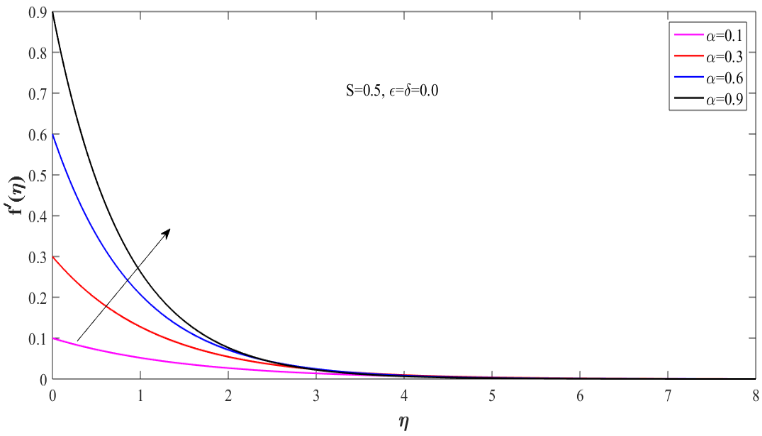

FIGURE 3.2: Variation in stretching parameter

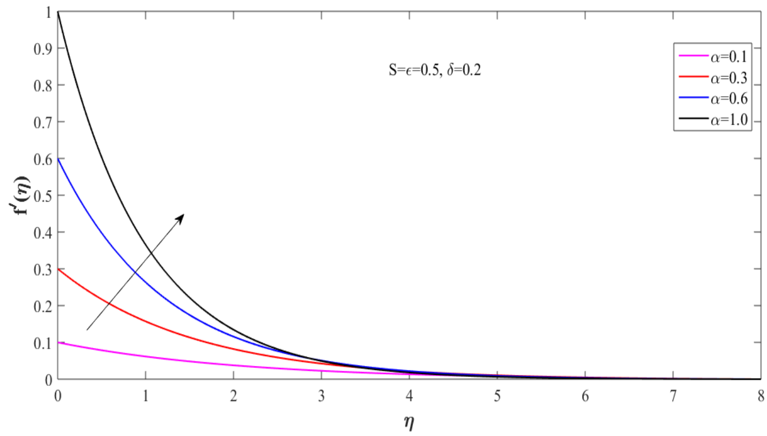

FIGURE 3.3: Variation in stretching parameter

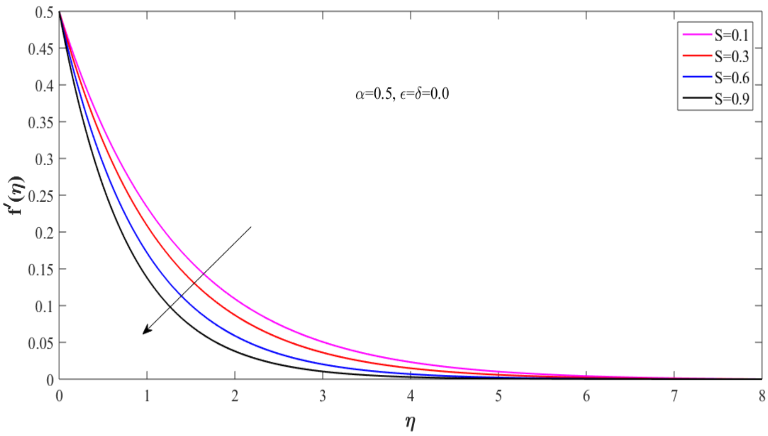

FIGURE 3.4: Variation in suction parameter

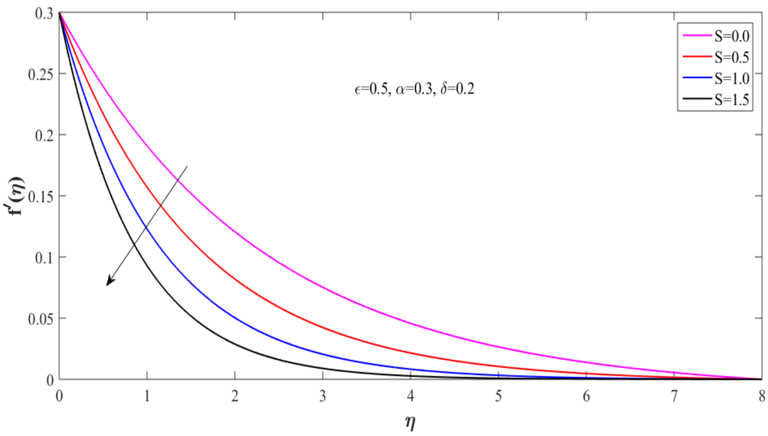

FIGURE 3.5: Variation in stretching parameter

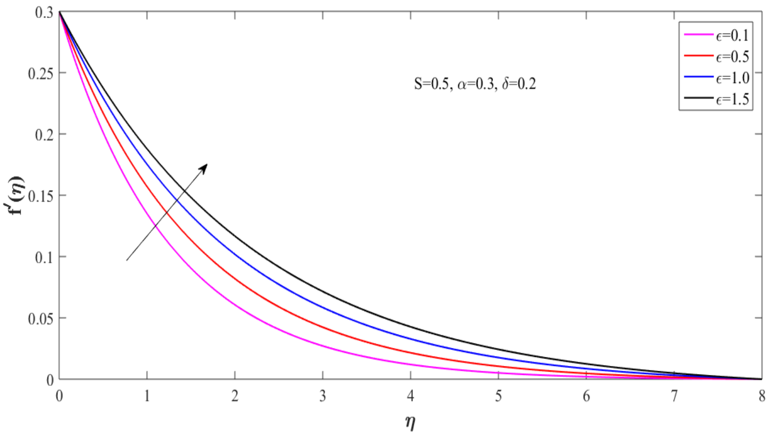

FIGURE 3.6: Variation in fluid parameter ε

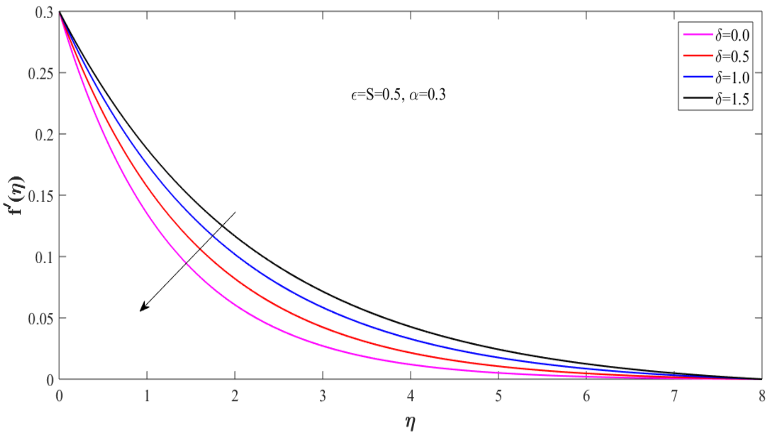

FIGURE 3.7: Variation in fluid parameter δ

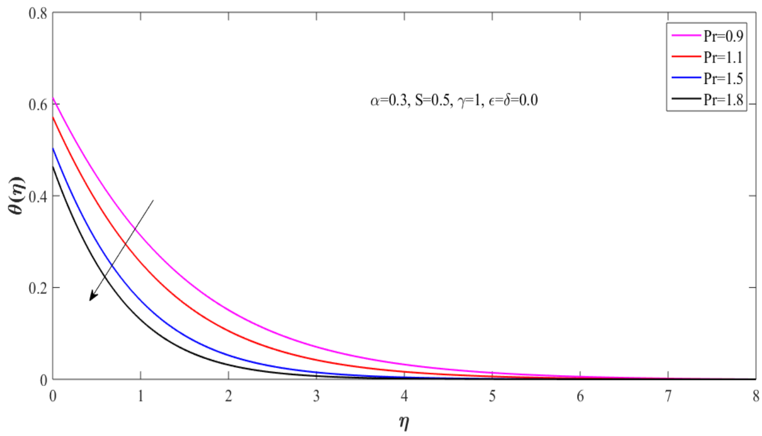

FIGURE 3.8: Variation in Prandtl number

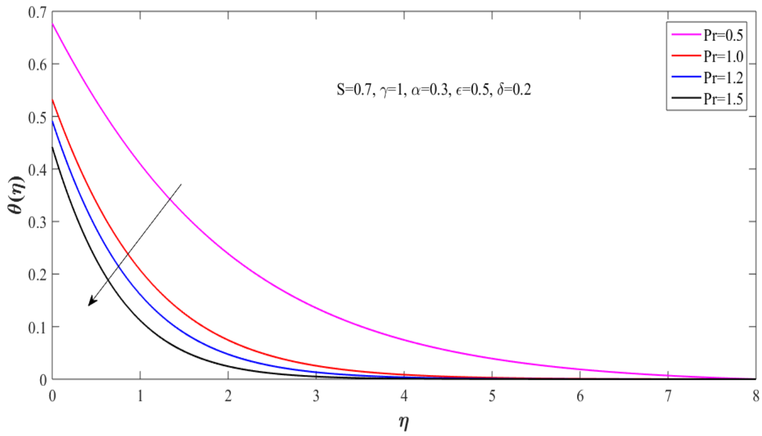

FIGURE 3.9: Variation in Prandtl number

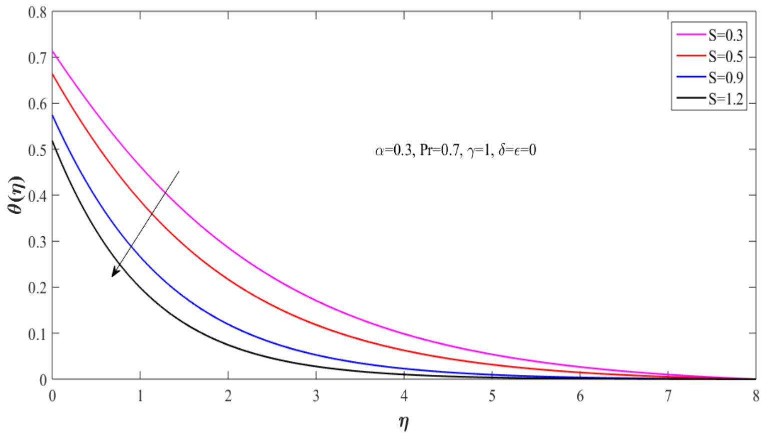

FIGURE 3.10: Variation in suction parameter

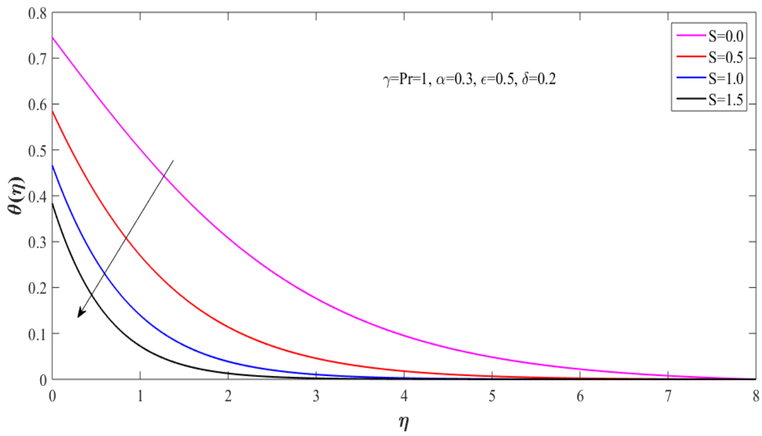

FIGURE 3.11: Variation in suction parameter

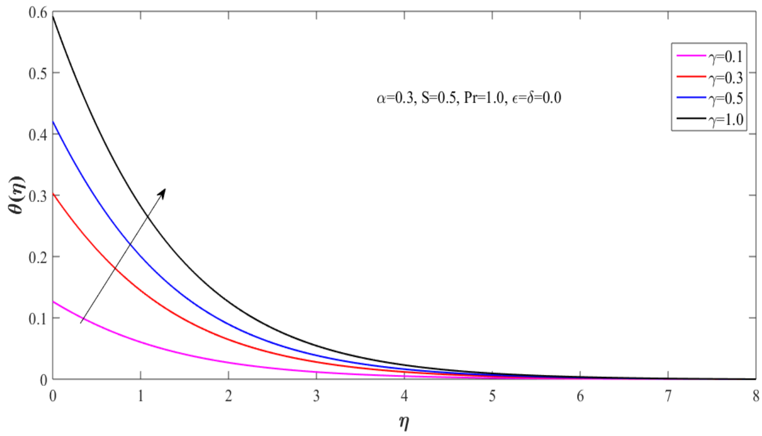

FIGURE 3.12: Variation in Biot number

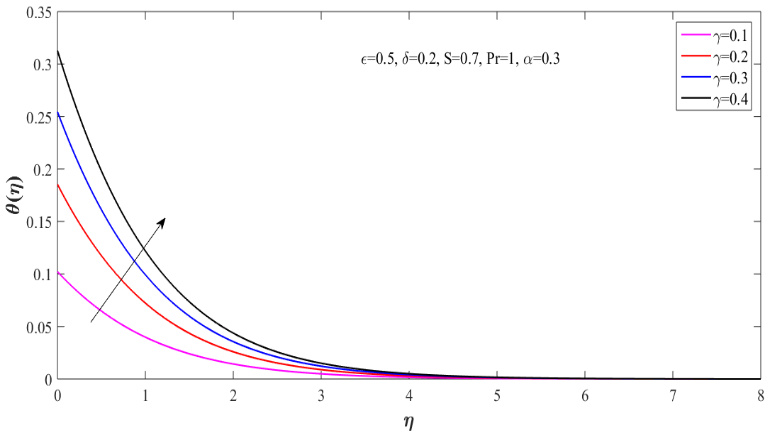

FIGURE 3.13: Variation in Biot number

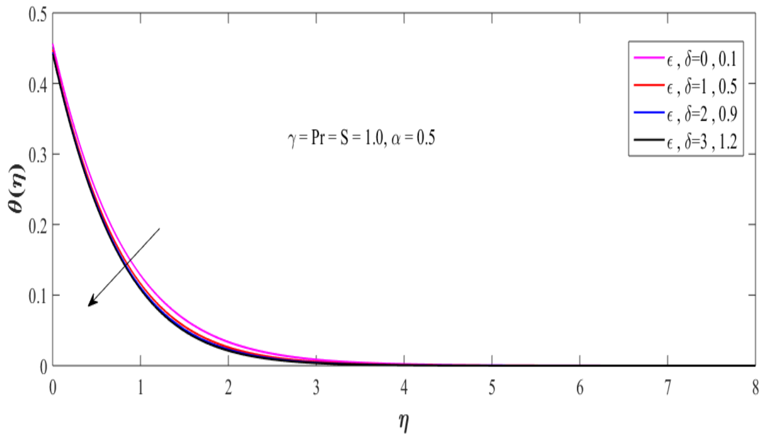

FIGURE 3.14: Variation in fluid parameters ε and δ

Chapter 4

MHD Flow of an Eyring-Powell Fluid with the Effect of Thermal Radiation and Joule Heating

4.1 Introduction

In the present chapter, the article of Hayat et al. [26] that was discussed in the previous chapter has been extended by considering the MHD boundary layer stream of the Eyring-Powell liquid on the extending sheet within the existence of warm radiation and Joule warming. By using the same similarity transformations already discussed in the previous chapter, we convert the non-linear PDEs into a framework of ODEs. Numerical solution of these resulting ODEs is obtained by the shooting technique utilizing MATLAB. At long last, the numerical comes about are examined at the conclusion of the chapter with a discourse on the impacts of diverse physical parameters.

4.2 Mathematical Modeling

A two-dimensional stream of an MHD Eyring-Powell liquid on a extending surface has been considered. The x and y axes are assumed to be along and opposite to the surface, respectively. The magnetic field B_0 acts in the y direction. The geometry of flow model is given in Figure 4.1.

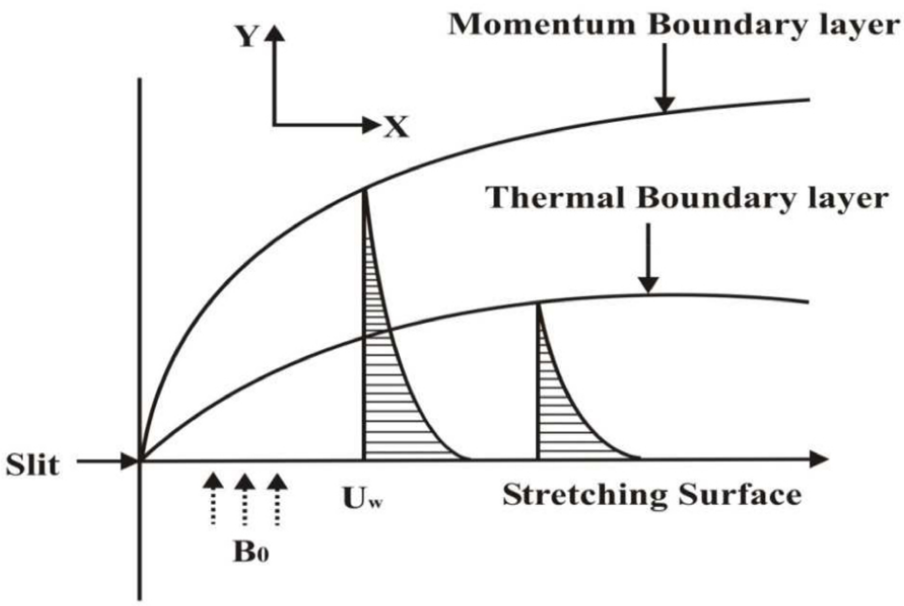

FIGURE 4.1: Geometry for the flow under consideration

The governing PDEs are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\nu + \frac{1}{\rho\beta C}\right)\frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho\beta C^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u, \tag{4.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\sigma B_0^2}{\rho c_p}u^2. \tag{4.3}$$

In the above equations, σ presents the conductivity of the fluid, and q_r represents the thermal radiation. The related boundary conditions for the over framework have been taken as:

$$\left.\begin{aligned} &u = u_w(x) = bx, v = v_w, -k\frac{\partial T}{\partial y} = h_f(T_f - T) \;\; at \;\; y = 0, \\ &u \to 0,\; T \to T_\infty \; as \; y \to \infty. \end{aligned}\right\} \tag{4.4}$$

The radiative heat flux q_r can be composed as

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{4.5}$$

where k^* is the mean retention coefficient and σ^* is the Stefan-Boltzman consistence.

Now T^4, the temperature variety, can be extended about T_∞ through the Taylor series, as follows:

$$T^4 = T_\infty^4 + \frac{4T_\infty^3}{1!}(T - T_\infty)^1 + \frac{12T_\infty^2}{2!}(T - T_\infty)^2 + \frac{24T_\infty}{3!}(T - T_\infty)^3 + \frac{24}{4!}(T - T_\infty)^4.$$

Ignoring the higher order terms,

$$\begin{aligned} & T^4 &&= T_\infty^4 + 4T_\infty^3(T - T_\infty) \\ \Rightarrow\;\; & T^4 &&= T_\infty^4 + 4T_\infty^3 T - 4T_\infty^4 \\ \Rightarrow\;\; & T^4 &&= 4T_\infty^3 T - 3T_\infty^4 \\ \Rightarrow\;\; & \frac{\partial T^4}{\partial y} &&= 4T_\infty^3 \frac{\partial T}{\partial y} \end{aligned} \tag{4.6}$$

Utilizing equation (4.6) in (4.5) and then differentiating w.r.t y, we get

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \tag{4.7}$$

Now the stream function ψ has been introduced which is assumed to satisfy the continuity equation as below:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}. \tag{4.8}$$

For the conversion of (4.1)-(4.3) into the dimensionless form, we present the taking after similitude change:

$$\psi = (a\nu)^{1/2} x f(\eta), \tag{4.9}$$

$$\eta = \left(\frac{a}{\nu}\right)^{1/2} y, \tag{4.10}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \tag{4.11}$$

$$T = (T_f - T_\infty)\theta(\eta) + T_\infty. \tag{4.12}$$

For the conversion of (4.1)-(4.3) into a system of ODEs, we use the above transformation. The continuity equation is satisfied as same way described in Chapter 3. Now, we include below the procedure for the conversion of momentum equation (4.2) into the dimensionless form.

- $$u\frac{\partial u}{\partial x} = axf'(\eta)af'(\eta) = a^2xf'^2(\eta). \tag{4.13}$$
- $$\frac{\partial u}{\partial y} = axf''(\eta)\frac{\partial \eta}{\partial y} = axf''(\eta)\left(\frac{a}{\nu}\right)^{1/2}.$$
- $$\begin{aligned} v\frac{\partial u}{\partial y} &= -(a\nu)^{1/2} f(\eta) axf''(\eta)\left(\frac{a}{\nu}\right)^{1/2}. \\ &= -a^2xf(\eta)f''(\eta). \end{aligned} \tag{4.14}$$

For the change of right side of (4.2) into the dimensionless frame, we continue as takes after.

- $$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(axf''(\eta)\left(\frac{a}{\nu}\right)^{1/2}\right) = x\frac{a^2}{\nu}f'''(\eta).$$
- $$\begin{aligned} \left(\nu + \frac{1}{\rho\beta C}\right)\frac{\partial^2 u}{\partial y^2} &= \left(\nu + \frac{1}{\rho\beta C}\right)\left(\frac{a^2}{\nu}xf'''(\eta)\right) \\ &= \left(1 + \frac{1}{\mu\beta C}\right)a^2xf'''(\eta). \end{aligned} \tag{4.15}$$
- $$\begin{aligned} \frac{1}{2\rho\beta C^3}\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2} &= \frac{1}{2\rho\beta C^3}\left(axf''(\eta)\left(\frac{a}{\nu}\right)^{1/2}\right)^2\left(\frac{a^2}{\nu}xf'''(\eta)\right) \\ &= \frac{1}{2\beta C^3\mu\nu}\left(a^5x^3(f''(\eta))^2f'''(\eta)\right). \end{aligned} \tag{4.16}$$

Utilizing (4.13)-(4.16) the final dimensionless form of (4.2) takes the form:

$$
\begin{aligned}
a^2x((f'(\eta))^2 - f(\eta)f''(\eta)) &= a^2x\Bigg(\left(1+\frac{1}{\mu\beta C}\right)f'''(\eta) \\
&\quad - \frac{1}{2\beta C^3\mu\nu}(a^3x^2(f''(\eta))^2f'''(\eta)) - \frac{\sigma B_0^2}{\rho a}f'(\eta)\Bigg) \\
\Rightarrow (f'(\eta))^2 - f(\eta)f''(\eta) &= \left(1+\frac{1}{\mu\beta C}\right)f'''(\eta) - \frac{1}{2\beta C^3\mu\nu}\left(a^3x^2(f''(\eta))^2f'''(\eta)\right) \\
&\quad - \frac{\sigma B_0^2}{\rho a}f'(\eta). \\
\Rightarrow (f'(\eta))^2 - f(\eta)f''(\eta) &= (1+\varepsilon)\,f'''(\eta) - \varepsilon\delta(f''(\eta))^2f'''(\eta) - M^2f'(\eta). \\
\Rightarrow\ (1+\varepsilon)\,f'''(\eta) - \varepsilon\delta(f''(\eta))^2f'''(\eta) &+ f(\eta)f''(\eta) - (f'(\eta))^2 - M^2f'(\eta) = 0.
\end{aligned}
\tag{4.17}
$$

Presently, we incorporate the below strategy for the transformation of (4.3) into the dimensionless shape:

- $$\frac{\partial T}{\partial x} = \theta'(\eta)(T_f - T_\infty)\frac{\partial \eta}{\partial x} = 0. \tag{4.18}$$
- $$u\frac{\partial T}{\partial x} = 0. \tag{4.19}$$
- $$\begin{aligned}\frac{\partial T}{\partial y} &= \theta'(\eta)(T_f - T_\infty)\frac{\partial \eta}{\partial y} \\ &= (T_f - T_\infty)\sqrt{\frac{a}{\nu}}\ \theta'(\eta).\end{aligned} \tag{4.20}$$
- $$\begin{aligned}v\frac{\partial T}{\partial y} &= -\sqrt{a\nu}(T_f - T_\infty)\sqrt{\frac{a}{\nu}}\ f\theta'(\eta). \\ &= -a(T_f - T_\infty)f\theta'(\eta).\end{aligned} \tag{4.21}$$
- $$\begin{aligned}\frac{\partial^2 T}{\partial y^2} &= (T_f - T_\infty)\sqrt{\frac{a}{\nu}}\frac{\partial \eta}{\partial y}\theta''(\eta) \\ &= \frac{a}{\nu}(T_f - T_\infty)\theta''(\eta).\end{aligned} \tag{4.22}$$

Using these in equation (4.3), we get

$$
\begin{aligned}
-a(T_f - T_\infty)f(\eta)\theta'(\eta) &= \frac{k}{\rho c_p}\frac{a}{\nu}(T_f - T_\infty)\theta''(\eta) + \frac{1}{\rho c_p}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{a}{\nu}(T_f - T_\infty)\theta''(\eta) \\
&\quad + \frac{\sigma B_0^2}{\rho c_p}(a^2x^2(f'(\eta))^2).
\end{aligned}
$$

$$\Rightarrow \; -f(\eta)\theta'(\eta) = \frac{k}{\rho c_p}\frac{1}{\nu}\theta''(\eta) + \frac{1}{\rho c_p}\frac{16\sigma^* T_\infty^3}{3k^*}\frac{1}{\nu}\theta''(\eta) + \frac{\sigma B_0^2}{\rho c_p(T_f - T_\infty)}(ax^2(f'(\eta))^2).$$

$$\Rightarrow \; -\frac{\nu}{k}\rho c_p f(\eta)\theta'(\eta) = \theta''(\eta) + \frac{16\sigma^* T_\infty^3}{3k^* k}\theta''(\eta) + \frac{\sigma B_0^2}{(T_f - T_\infty)}\nu a x^2 (f'(\eta))^2.$$

$$\Rightarrow \; -Prf(\eta)\theta'(\eta) = \theta''(\eta) + \frac{4}{3}R_d\theta''(\eta) + M^2 PrEc(f'(\eta))^2.$$

$$\Rightarrow \left(1 + \frac{4}{3}R_d\right)\theta''(\eta) + Prf(\eta)\theta'(\eta) + M^2 PrEc(f'(\eta))^2 = 0.$$

The final dimensionless form of the given model is

$$(1+\varepsilon) f''' - \varepsilon\delta f''^2 f''' + ff'' - f'^2 - M^2 f' = 0. \tag{4.23}$$

$$\left(1 + \frac{4}{3}R_d\right)\theta'' + Prf\theta' + M^2 PrEcf'^2 = 0. \tag{4.24}$$

The associated boundary conditions (4.4) take the form:

$$\left.\begin{aligned} &f(\eta) = S,\; f'(\eta) = \alpha,\; \theta'(\eta) = -\gamma(1-\theta(\eta)) \;\text{ at } \eta = 0,\\ &f'(\eta) \to 0,\; \theta(\eta) \to 0 \;\text{ as } \eta \to \infty. \end{aligned}\right\} \tag{4.25}$$

Different parameters used in the above equations have the following formulation:

$$\left.\begin{aligned} &\varepsilon = \frac{1}{\mu\beta C},\quad \delta = \frac{a^3x^2}{2C^2\nu},\quad Pr = \frac{\mu c_p}{k},\quad M^2 = \frac{\sigma B_0^2}{\rho a},\quad R_d = \frac{4\sigma^* T_\infty^3}{k^* k}\\ &S = -\frac{v_w}{\sqrt{a\nu}},\quad \alpha = \frac{b}{a},\quad \gamma = \frac{h_f}{k}\sqrt{\frac{\nu}{a}},\quad Ec = \frac{u_w^2}{C_p(T_w - T_\infty)}. \end{aligned}\right\} \tag{4.26}$$

The surface drag coefficient, is characterized as:

$$C_{fx} = \frac{\tau_w}{\rho u_w^2}, \tag{4.27}$$

where the wall skin friction is given by

$$\tau_w = \left(\mu + \frac{1}{\beta C}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0} - \frac{1}{6\beta}\left(\frac{1}{C}\left(\frac{\partial u}{\partial y}\right)_{y=0}\right)^3.$$

The surface drag coefficient within the dimensionless shape is

$$(Re_x)^{1/2}\, C_f x = (1+\varepsilon) f''(0) - \frac{1}{3}\varepsilon\delta(f''(0))^3, \tag{4.28}$$

where the Reynolds number is given as

$$Re_x = \frac{ax^2}{\nu}.$$

The Nusselt number is characterized by:

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \tag{4.29}$$

where

$$q_w = -\left(k + \frac{16\sigma^* T_\infty^3}{3k^*}\right)\left(\frac{\partial T}{\partial y}\right)_{y=0}.$$

The Nusselt number is obtained as

$$\begin{aligned}
Nu_x &= -\frac{\left(k + \frac{16\sigma^* T_\infty^3}{3k^*}\right) x(T_f - T_\infty)\sqrt{\frac{a}{\nu}}\theta'(0)}{k(T_w - T_\infty)} \\
&= -x\sqrt{\frac{a}{\nu}}\left(\frac{k}{k} + \frac{16\sigma^* T_\infty^3}{3k^* k}\right)\theta'(0) \\
&= -(Re_x)^{1/2}\left(1 + \frac{4}{3}R_d\right)\theta'(0). \\
\Rightarrow (Re_x)^{-1/2} Nu_x &= -\left(1 + \frac{4}{3}R_d\right)\theta'(0).
\end{aligned}$$

4.3 Numerical Treatment

In our work we used shooting method to solve the transformed ODEs subject to the boundary condition. For the implementation of the shooting method we first convert the system of equations into first order ODEs.
Let us use the notations:

$$\begin{aligned}
&f = y_1,\ f' = y_1' = y_2,\ f'' = y_2' = y_3, \\
&\theta = y_4,\ \theta' = y_4' = y_5.
\end{aligned}$$

The system of equations (4.23) and (4.24) along with the initial conditions from (4.25), can presently be composed within the frame of the first order ODEs

$$
\begin{aligned}
y_1' &= y_2, && y_1(0) = S,\\
y_2' &= y_3, && y_2(0) = \alpha,\\
y_3' &= \frac{M^2 y_2 + y_2^2 - y_1 y_3}{1+\varepsilon - \varepsilon\delta y_3^2}, && y_3(0) = p,\\
y_4' &= y_5, && y_4(0) = q,\\
y_5' &= -\frac{1}{(1+\frac{4}{3}R_d)} Pr(y_1 y_5 + M^2 Pr Ec y_2^2), && y_5(0) = -\gamma(1-q).
\end{aligned}
$$

Because the numerical computations can not be performed on an unbounded domain, therefore the domain of the above problem has been taken as $[0, \eta_\infty]$ instead of $[0, \infty)$, where η_∞ is a finite positive number for which the variations in the solution are negligible after $\eta = \eta_\infty$. The ideal missing conditions p and q are assumed to satisfy the following relations.

$$
y_2(\eta_\infty, p, q) = 0, \tag{4.30}
$$

$$
y_4(\eta_\infty, p, q) = 0. \tag{4.31}
$$

The system of equations (4.30)-(4.31) will be solved by the Newton's method governed by the following formulation.

$$
\begin{pmatrix} p^{(k+1)} \\ q^{(k+1)} \end{pmatrix} = \begin{pmatrix} p^k \\ q^k \end{pmatrix} - \begin{pmatrix} \frac{\partial y_2}{\partial p} & \frac{\partial y_2}{\partial q} \\ \frac{\partial y_4}{\partial p} & \frac{\partial y_4}{\partial q} \end{pmatrix}^{-1}_{(p^k,\ q^k,\ \eta_\infty)} \begin{pmatrix} y_2 \\ y_4 \end{pmatrix}_{(p^k,\ q^k,\ \eta_\infty)}.
$$

The following new variables have been introduced for the further proceedings.

$$
\begin{aligned}
&\frac{\partial y_1}{\partial p} = y_6, \quad \frac{\partial y_2}{\partial p} = y_7, \quad \frac{\partial y_3}{\partial p} = y_8, \quad \frac{\partial y_4}{\partial p} = y_9, \quad \frac{\partial y_5}{\partial p} = y_{10},\\
&\frac{\partial y_1}{\partial q} = y_{11}, \quad \frac{\partial y_2}{\partial q} = y_{12}, \quad \frac{\partial y_3}{\partial q} = y_{13}, \quad \frac{\partial y_4}{\partial q} = y_{14}, \quad \frac{\partial y_5}{\partial q} = y_{15}.
\end{aligned}
$$

By inserting these notations in the above Newton's iterative formulation, we get the form:

$$\begin{pmatrix} p^{(k+1)} \\ q^{(k+1)} \end{pmatrix} = \begin{pmatrix} p^{k} \\ q^{k} \end{pmatrix} - \begin{pmatrix} y_7 & y_{12} \\ y_9 & y_{14} \end{pmatrix}^{-1}_{(p^k,\ q^k,\ \eta_\infty)} \begin{pmatrix} y_2 \\ y_4 \end{pmatrix}_{(p^k,\ q^k,\ \eta_\infty)}.$$

Now differentiate the above system of ODEs, first w.r.t p and then w.r.t q. The complete IVP can be written as:

$$\begin{aligned}
&y_7' = y_8, && y_7(0) = 0,\\
&y_8' = \frac{M^2 y_2 + y_2^2 - y_1 y_3}{(1+\varepsilon-\varepsilon\delta y_3^2)^2}(2\delta\varepsilon y_3 y_8) + \frac{M^2 y_7 + 2y_2 y_7 - y_1 y_8 - y_6 y_3}{1+\varepsilon-\varepsilon\delta y_3^2}, && y_8(0) = 1,\\
&y_9' = y_{10}, && y_9(0) = 0,\\
&y_{10}' = -\frac{1}{(1+\frac{4}{3}R_d)} Pr(y_6 y_5 + y_1 y_{10} + 2M^2 Ec y_2 y_7), && y_{10}(0) = 0,\\
&y_{11}' = y_{12}, && y_{11}(0) = 0,\\
&y_{12}' = y_{13}, && y_{12}(0) = 0,\\
&y_{13}' = \frac{M^2 y_2 + y_2^2 - y_1 y_3}{(1+\varepsilon-\varepsilon\delta y_3^2)^2}(2\delta\varepsilon y_3 y_{13}) + \frac{M^2 y_{12} + 2y_2 y_{12} - y_1 y_{13} - y_{11} y_3}{1+\varepsilon-\varepsilon\delta y_3^2}, && y_{13}(0) = 0,\\
&y_{14}' = y_{15}, && y_{14}(0) = 1,\\
&y_{15}' = -\frac{1}{(1+\frac{4}{3}R_d)} Pr(y_{11} y_5 + y_1 y_{15} + 2M^2 Pr Ec y_2 y_{12}), && y_{15}(0) = \gamma.
\end{aligned}$$

The Newton's iterative process is repeated up till the following condition is met.

$$max\{|y_2(\eta_\infty)|\ ,\ |y_4(\eta_\infty)|\} \le \xi,$$

where ξ is a small positive number. For the computational purpose, ξ has been given the value $\xi = 10^{-8}$ whereas η_∞ is set as 8.

4.4 Tabular and Graphical Results

Table 4.1 shows the behaviors of ε, δ, Pr, S, γ, M, Ec and R_d on the surface drag coefficient and the rate of heat transfer. By increasing the values of ε both the surface drag coefficient and the rate of heat transfer increase. Increasing the values of δ, the surface drag coefficient and the rate of heat transfer are observed to decrease. Increasing the values of Pr gives no effect on the the surface drag coefficient while the rate of heat transfer increases with an increment in the Prandtl number. By increasing S, there is also an increase in the values of drag coefficient and heat exchange rate. By increasing the Biot number γ, there is no effect on the drag coefficient but heat exchange rate goes to increase. By increasing the magnetic parameter M, the values of surface drag increases while those of rate of heat exchange decrease with an increment in M. By increasing the Eckert number there is a decrement in the values of drag coefficient. Increasing R_d also gives increase in the values of heat exchange.

ε	δ	Pr	S	γ	M	Ec	R_d	$Re_x^{1/2}C_f$	I_f	$Re_x^{-1/2}Nu_x$
0.2	0.2	1.0	0.5	1.0	0.2	0.1	0.1	0.536155	[-0.5, 0.2]	0.468495
0.3								0.551442	[-0.5, 0.1]	0.4413199
0.5								0.580590	[-0.5, 0.1]	0.4442502
1.0								0.646956	[-0.5, 0.2]	0.4494621
0.3	0.1							0.551591	[-0.4, 0.1]	0.4412674
	0.5							0.550992	[-0.4, 0.1]	0.4412674
	1.0							0.550233	[-0.4,-0.1]	0.4411786
	0.2	0.5						0.551442	[-0.5, 0.1]	0.4791642
		1.2						0.551442	[-0.5, 0.1]	0.4791642
		1.5						0.551442	[-0.5, 0.1]	0.5257594
		1.0	0.2					0.461270	[-0.4,-0.1]	0.3608930
			1.0					0.727692	[-0.6, 0.2]	0.5472029
			1.0	0.5				0.727692	[-0.6, 0.2]	0.3536723
				1.5				0.727692	[-0.6, 0.2]	0.6692802
				1.5	0.3			0.732327	[-0.6, 0.1]	0.7089381
					0.5			0.746948	[-0.7,-0.1]	0.7059973
					0.5	0.2		0.746948	[-0.7,-0.1]	0.7045953
						0.5		0.746948	[-0.7,-0.1]	0.7003892
						0.5	0.2	0.746948	[-0.7,-0.1]	0.6588922
							0.3	0.746948	[-0.7,-0.1]	0.6223241

TABLE 4.1: Numerical outcomes of $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ for distinctive parameters when α =0.5

Figure 4.2 describes that by expanding the values of the liquid parameter ε, there's an increase within the boundary layer thickness of the speed profile. Figure 4.3 shows the impact of the extending proportion parameter α on the speed and the boundary layer thickness which is related to it.It highlights the impact of the stretching ratio parameter α on the velocity profile. By this figure, it is clear that increasing α guides to an increment in the density of the velocity boundary layer. Figure 4.4 describes the impact of the suction parameter S on the speed $f'(\eta)$. The suction parameter S and the speed $f'(\eta)$ are noticed to have an inverse connection. An increase within the values of S causes a diminishment within the boundary layer thickness of the speed. Figure 4.5 describes the impact of M on the speed profile which demonstrates that by expanding the values of M, the speed goes to diminish and the boundary layer thickness could be a diminishing function of M.

Figures 4.6-4.12 show the effects of distinctive parameters, ε, α, S, M, Pr, Ec, R_d and γ on the temperature. Figure 4.6 gives the impact of ε on the temperature. By expanding the characters of this material fluid parameter, the temperature goes to decrease. It has an inverse relation with the temperature. Figure 4.7 shows the influence of the suction parameter S on the temperature distributions. The temperature goes to decrease as we increase the values of S. Figure 4.8 shows the effect of M on the temperature distribution. By expanding the values of M, the temperature profile increases. From figure 4.9, it is observed that as we increase the values of Prandtl number, the thermal diffusivity starts decreasing. So the temperature and the thermal boundary layer thickness are the decreasing functions of Prandtl number. Figure 4.10 and 4.11 show the influence of Eckert number and radiation parameter R_d on the temperature distribution. It is observed that there is an increase in the temperature profile by increasing the values of Ec and R_d. These are increasing functions of temperature. Figure 4.12 indicates the impact of Biot number γ on temperature. From this figure, it is seen that as we increase the values of Biot number, the rate of heat transfer also increases. So for an increment in the heat transfer rate, the temperature also increases.

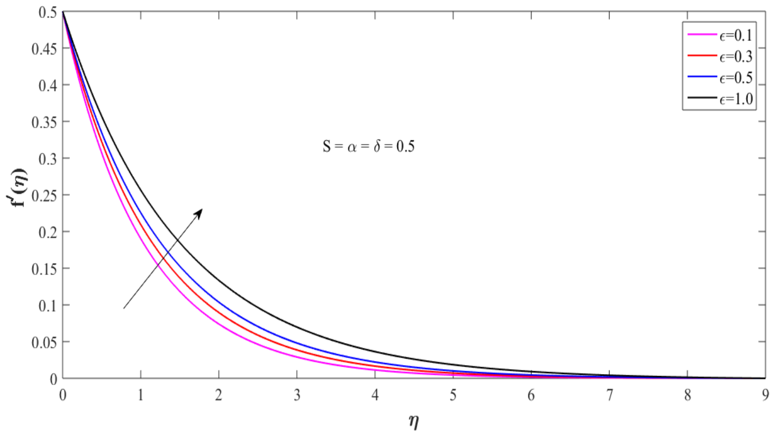

FIGURE 4.2: Effect of ε on $f'(\eta)$

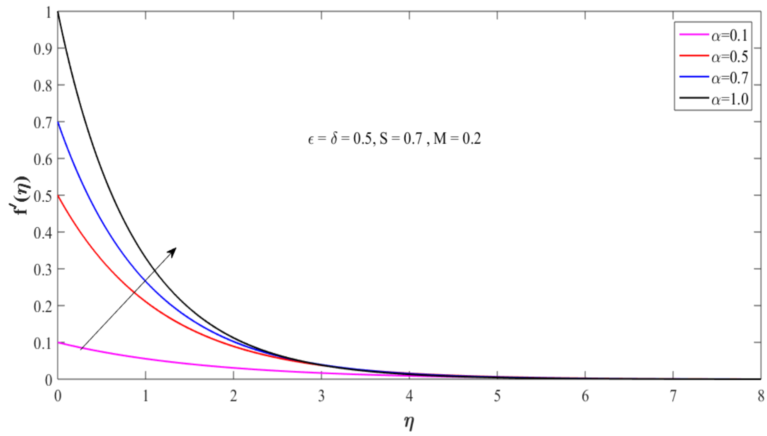

FIGURE 4.3: Effect of α on $f'(\eta)$

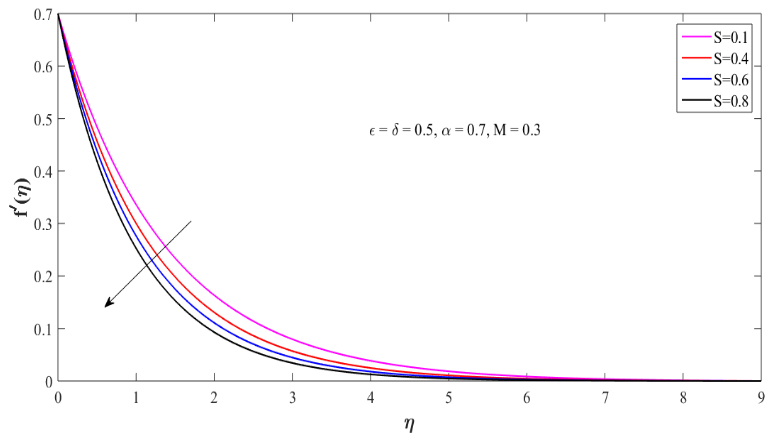

FIGURE 4.4: Effect of S on $f'(\eta)$

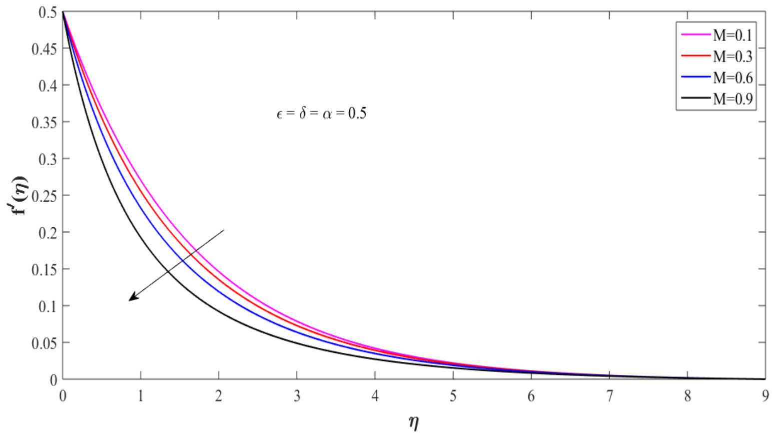

FIGURE 4.5: Effect of M on $f'(\eta)$

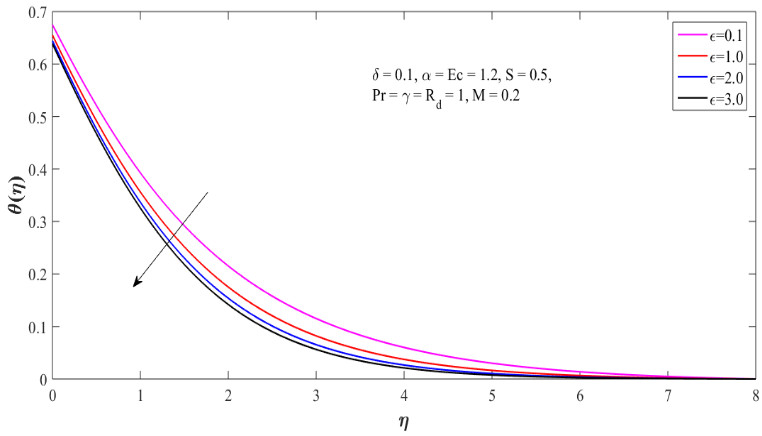

FIGURE 4.6: Effect of ε on $\theta(\eta)$

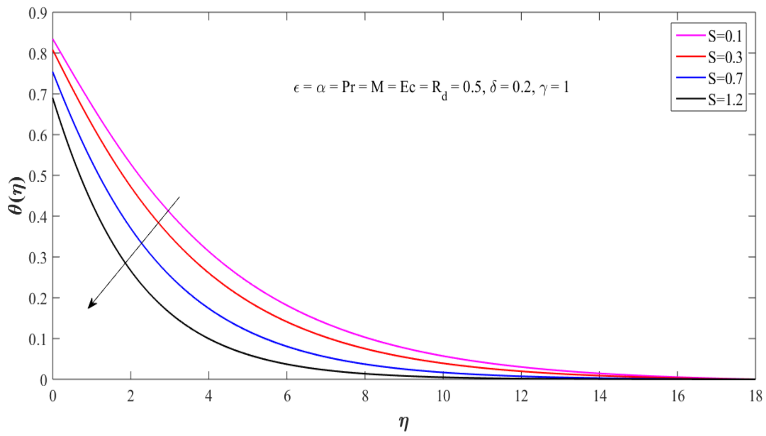

FIGURE 4.7: Effect of S on $\theta(\eta)$

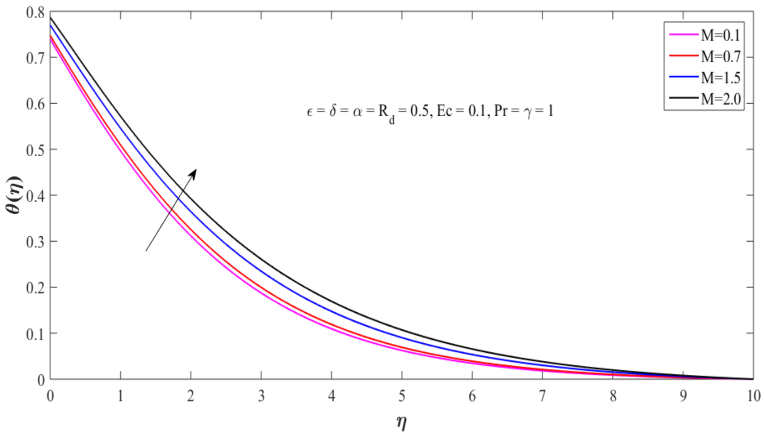

FIGURE 4.8: Effect of M on $\theta(\eta)$

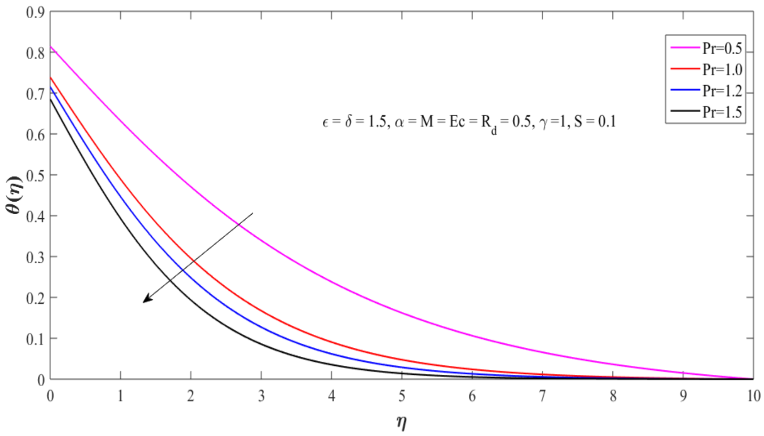

FIGURE 4.9: Effect of Pr on $\theta(\eta)$

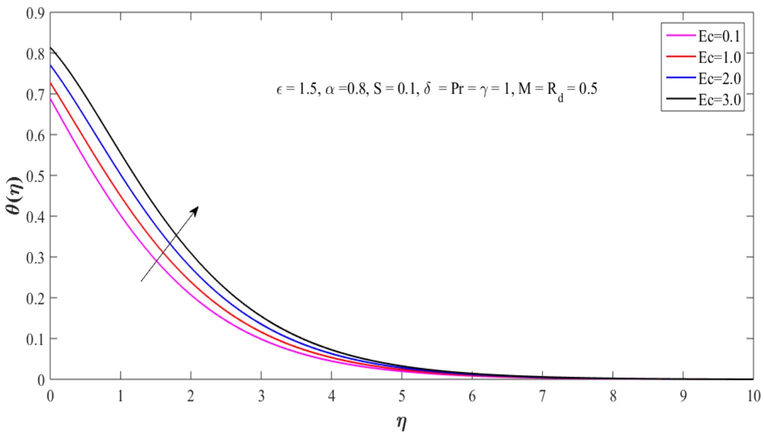

FIGURE 4.10: Effect of Ec on $\theta(\eta)$

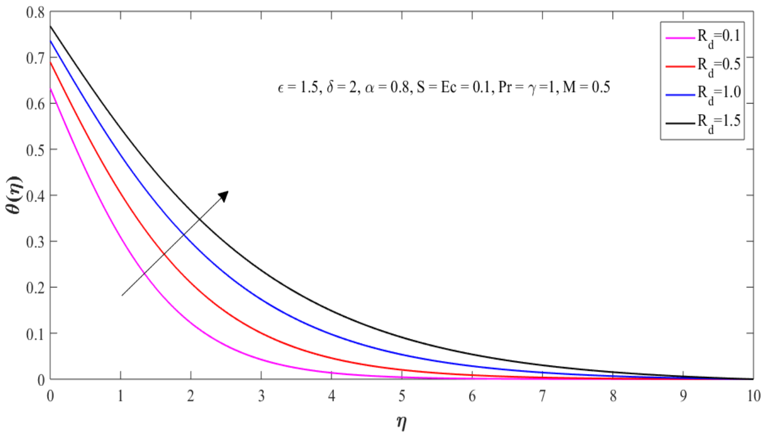

FIGURE 4.11: Effect of R_d on $\theta(\eta)$

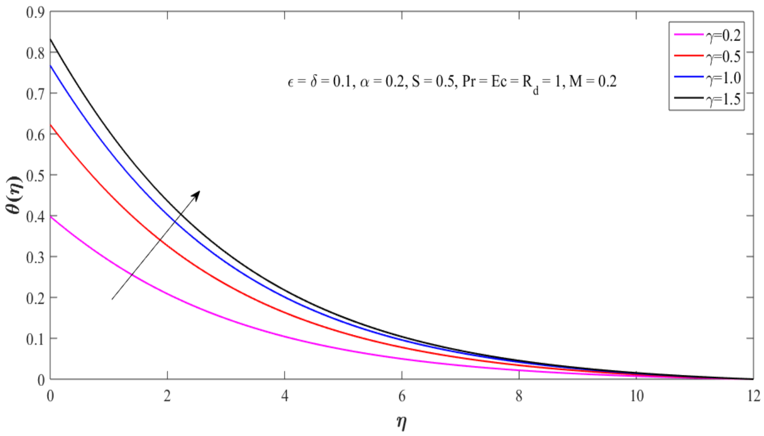

FIGURE 4.12: Effect of γ on $\theta(\eta)$

Chapter 5

Conclusion

Summary of this research work represents the analysis of two-dimensional magnetohydrodynamics flow of the Eyring-Powell fluid under the impact of joule heating and thermal radiation along a stretchable surface. The governing nonlinear PDEs of momentum and energy are changed into the ODEs by utilizing a proper similarity transformation. Numerical solution of the mathematical model was carried out by using the shooting technique. Significance of the effects of different physical parameters under discussion on the dimensionless velocity and temperature are delineated graphically. The skin friction coefficient and the Nusselt number for different value of the distinctive governing parameters are presented in the tabular form. After a thorough investigation, we have reached the following concluding observation.

- Increasing the values of suction parameter (S), the velocity profile and the temperature profile tends to increase.
- Increased velocity profile while reduced temperature plots has been discovered for the rising values of fluid material parameter (ε).
- Increasing the values of the Prandtl number, the temperature is seen to decline.

- The magnetic field M has a direct relation with the temperature profile and an inverse with the velocity profile.
- Increasing the thermal radiation, the temperature tends to rise.
- The skin friction is increasing for the increment of ε and decreasing for the increment of δ.
- The local Nusselt number is increasing function of R_d, S, δ, Pr and decreasing for ε, M and Ec.

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